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EFFECTS OF BACKLASH IN THE SECOND ORDER SERVO

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EFFECTS OF BACKLASH IN THE SECOND ORDER SERVO

by

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ABSTRACT

By means of an automatic digital computer solutions were obtained for a number of second order servo problems with backlash in the gear train. A complete range of solutions were obtained with variations in load friction, motor friction, load inertia, motor inertia, damping coefficient for the combined operation, and backlash angle. These data are presented in chart form suitable for the servo design engineer to estimate performance of an arbitrary second order system with backlash.

The program was conducted on the Control Data Corporation Model 1604 Computer, and this program is presented with explanations of the manner in which the differential equations were solved. From data generated by the computer phase plane plots are drawn as a further illustration of the results of this thesis, and a sample problem is included to show application of the design charts.

The writer wishes to express his appreciation to Dr. George J. Thaler of the Electrical Engineering Department and to Dr. E. J. Stewart and Mr. E. N. Ward of the U. S. Naval Postgraduate Computer Center for their assistance and encouragement in completing this work.

TABLE OF CONTENTS

1. Introduction

In the engineering analysis and application of servomechanisms (hereafter called servos) which include gear coupling, the problem of backlash must be considered and is often a source of trouble. In theory it is desirable to know the system performance of a servo with a given distribution of inertia and friction and to accurately determine the size of the limit cycle for any given set of variables. In practice, knowledge of the performance of a system may be desirable prior to the construction of a mock-up or simulation system.

Existing methods for the analytical solution of the generalized backlash problem are laborious and complicated and lack of the straightforward approach needed in the preliminary design of a servo. In this thesis design charts are presented for the second order servo with backlash to accommodate an arbitrary set of system variables.

Previous work in this problem has been done at the U. S. Naval Postgraduate School by Lutkenhouse in reference a wherein the backlash problem was solved by construction of the phase trajectory in the graphical phase plane. This thesis extends the work of Lutkenhouse. It is recognized that the construction of a phase trajectory for a given problem requires many hours of tedious calculation and graphical construction. Thus, in the collection of a large volume of data required for the construction of design charts for the second order servo with backlash an automatic digital computer was employed.

The work of this thesis first required derivation of the system equations, then a computer program was developed to adequately describe the equations. The burden of the work subsequently fell in processing sufficient data to construct design charts to satisfy the requirements of the problem.

2. Equations for the Servo Motion

In the construction of a set of design charts it is usually desirable to provide sufficient flexibility so that the charts may be used for any given servo. Thus, equations are presented in Appendix A for the determination of the system parameters necessary to enter the design charts of this thesis for three types of second order servos: the armature control motor, the field control motor, and the two phase motor.

All of these cases behave in a similar manner when coupled to a load through a gearbox which has backlash. Initially there is a period when the backlash is taken up, and the system performs as a combined system. The differential equation describing this motion for an undamped natural frequency of one is

$$\ddot{\theta} + B\dot{\theta} + \theta = 1$$

where B is equal to twice the damping coefficient, ζ .

At such point in the cycle of motion wherein the load velocity exceeds that of the motor, backlash starts, and the motor is not directly coupled to the load. There are two differential equations describing this motion. The equation for the coasting load is

$$\ddot{\theta}_L + D\dot{\theta}_L = 0$$

where $D = \frac{F_L}{J_L}$

The equation for the motor is

$$E\ddot{\theta}_M + F\dot{\theta}_M + \theta_L = 1$$

where $E = J_M$ and $F = F_M$.

This motion in the backlash region continues until the total amount of backlash is taken up, and at that point gear contact is made. The action of the servo upon contact is not a simple one. This motion can follow one of several types of contact where the basic laws governing the motion are the law of conservation of momentum, and the law of conservation of energy which must both be considered. In this thesis it is assumed that the contact is a plastic one with no gear bounce; therefore, there is energy loss in the gearbox due to the plastic contact and the law of conservation of momentum was applied. Following the gear contact phase the system is again combined, and the cycle of events begins again.

The characteristics of the second order servo with backlash is independent of the undamped natural frequency as shown by the development in Appendix A. Therefore, all of the problems worked in this thesis and used in the construction of design charts are based on an undamped natural frequency of one radian per second, and the step input is equal to one radian. Units of radians and seconds are used throughout.

The amount of backlash used in the program was 0.3 radian, and it was a purpose of the thesis to determine the effect of other magnitudes of backlash.

3. Solution of the Equations

The differential equations for the second order servo with backlash were solved on the Control Data Corporation Model 1604 Computer. This computer is described in reference b, but briefly, it is an all-transistorized, stored program, general-purpose digital computer having a large storage capacity and exceedingly fast computation and transfer speeds. Input to the computer was by means of magnetic tape and punched paper tape, and the output data was recorded on magnetic tape. The data was then printed on the International Business Machine 717 high speed printer.

Figure 1 shows a schematic of the problem. In programming for the solution of the equations it was necessary to successively compute a point for the combined motion, test this point to determine if the load would drift away from the motor, then if so, continue the cycle into the backlash region. A second test was then introduced to determine the end of the backlash region, and if the backlash was fully taken up the contact part of the program was initiated. This complete program was accomplished by the program of Table I.

Several instructions in the program are worthy of mention. First, it is noted that a subroutine was used in the actual solution of the differential equations. This subroutine based on the Runge-Kutta Gill numerical integration is contained in a library tape in the U. S. Naval Postgraduate Computer Center, and programming instructions are contained in reference c. The program was assembled in accordance with the "AR" assembly routine of references d and e which also contain a full description of each instruction used in this program.

In the Runge-Kutta Gill numerical integration four successive approximations are used in the computation of a point (in this thesis points are 0.01 second apart). However, it was not desired to print out each point, so a count routine was inserted in the program to print each eighth point. Thus in the actual printout, an example of which is contained in Table II, test points are at intervals of .08 seconds. The printout format shows the number of the point in column 1, the time in column 2, the motor speed in column 3, the motor position in column 4, the load speed in column 5, and the load position in column 6. The number following the figure printed represents a decimal exponent to locate the decimal point.

The remaining item of the program requiring explanation is the portion of the program named switch. This routine is inserted to accommodate the various changes in sign caused by the backlash non-linearity. A close study of this sub-routine will show that it was not only necessary to change the sign of some of the instructions, but it was also necessary to change some of the jump instructions caused by multiple sign changes in the backlash test.

With the basic program of Table I stored in the computer, it is merely necessary to insert the desired system parameters for the particular problem to be solved into the computer locations beginning at 20156 in order to obtain the solution for any desired servo problem with backlash.

4. Results

A total of 120 runs for the second order servo with backlash were made and the limit cycle for these problems were determined. Each run required approximately three minutes on the computer and yielded one test point suitable for presenting in this report. These test points are plotted in Figures 2 through 11. Figures 12 through 15 are phase trajectories for a number of selected runs plotted from an actual printout.

Figure 2 is a plot of limit cycle versus friction ratio for a damping coefficient of 0.1 and inertia ratios of 1/9, 1/4, 1, 4 and 9.

Figure 3 is a plot of limit cycle versus friction ratio for the load inertia equal to the motor inertia and for damping coefficients of 0.1, 0.2, 0.3, 0.4, 0.5, and 0.6.

Figure 4 is a plot of limit cycle versus backlash angle for load inertia equal to motor inertia and for a friction ratio of 0.2. This plot shows a linear variation for damping coefficients of 0.1, 0.2, 0.3, 0.4, 0.5, and 0.6.

Figure 5 is a plot of limit cycle versus backlash angle for load inertia equal to motor inertia and for a damping coefficient equal to 0.1. This line represents all friction ratios since the $J_M = J_L$ variation of Figure 2 is a non-varying straight line. For a damping coefficient other than 0.1 the curve would spread to form a pencil of lines.

Figure 6 is a plot of limit cycle versus backlash angle for a damping coefficient of 0.1 and for a friction ratio of 0.2. Variations of the curves of this show inertia ratios of 1/4, 1 and 4.

Figures 7 through 11 are plots of limit cycle versus inertia ratios for friction ratios of 0.1, 0.2, 0.4, 0.6, and 1.0. Each plot shows curves for variations of damping coefficient of 0.1 through 0.6.

Figures 12 through 15 are actual plots of the phase trajectories as plotted from the print-out sheets. From these plots can be determined the output speed and position with the corresponding time.

Figure 12 shows a case with friction ratio of 0.2 at an inertia ratio of .25 and a damping coefficient of 0.1. Figure 13 is at the same friction ratio of 0.2 but shows an inertia ratio of 4 and a damping coefficient of 0.6. Figure 14 and 15 illustrates the effect of increasing the damping coefficient from 0.1 to 0.2 with load inertia equal to motor inertia and at a friction ratio of 1.0.

5. Discussion

After the accumulation of the data it was desirable to present the information in the most useful and practical form. The size of the limit cycle as shown in the figures of this thesis was previously defined as deviation from the equilibrium or average value rather than peak to peak variation. The data were plotted in various forms in an attempt to find the most useful form. Figure 2 is one such presentation where the data is plotted for constant damping coefficient versus friction ratio. Figure 2 was not found to be the optimum form because in subsequent plots at different values of damping coefficient there was appreciable crossing and recrossing of lines of constant inertia ratio, and it was found that interpolation between these lines was difficult. Further, there was not a wide variation of limit cycle with friction ratio as evidenced by the level curves of Figure 2. In fact, for the particular case of balanced load and motor inertia and damping coefficient there was no variation in limit cycle with friction ratio. This plot is shown as a logarithmic plot because it was found that the digital program required a number of modifications to satisfy the zero friction case. A single case was run with zero friction ratio, and the limit cycle was found to be substantially equal to that for a friction ratio of 0.04 which is used in this thesis as representative of the zero friction ratio case.

Figure 3 is a composite plot showing all values of damping coefficient up to 0.6. Figures 4, 5, and 6 are presented to show that the size of the limit cycle is directly proportional to the amount of backlash.

Several runs were made with changes in only the backlash to conclusively prove this characteristic. Figure 4 shows the variation with damping co efficient with all other variables constant. Figures 5 and 6 are similar plots with variations in other factors. This linear variation of limit cycle with backlash is significant since it means that design charts can be drawn for any constant amount of backlash and variation from this value is in direct proportion. In this thesis all charts are for a constant amount of backlash equal to 0.3 except for those points in Figures 4, 5, and 6 where backlash in a variable.

Figures 7 through 11 are the design charts found to be the most useful form for design estimations. In these charts the size of the limit cycle is plotted versus inertia ratio. The logarithmic scale was chosen for two reasons: first, it avoided the zero cases found at the extreme ends of the scales and for which a separate program would have been required, and secondly, this scale gives a spread to the curves in the region of inertia ratio equal to one which appears desirable.

The design charts presented herein are most useful for the range of variables encompassed by the test points. While it is believed that some estimates can be made from these charts for systems with very large and very small inertia ratios and beyond the limits of the curves shown, best results are obtained for values where interpolation between the lines of constant damping coefficient is feasible. Appendix B gives a sample problem using the design charts. Appendix C gives instructions in the use of the design charts.

6. Conclusions

From the results obtained in the accomplishment of this thesis it is concluded that the size of the limit cycle of a second order servo with backlash can be represented by design charts, one form of which are presented in Figures 7 through 11 of this thesis.

7. Recommendations

Inasmuch as it was necessary to complete this thesis during a Summer period, time was of essence, and the scope of the program was definitely limited. Therefore, the following recommendations are submitted as a reasonable extention of the program.

1. Rewrite the digital program to include a test to determine the actual size of limit cycle for display on the console with no printout required. This will permit longer runs with no time required for printout of data. Printout capability should still be retained by means of a Jump Key.

2. Extend the program to include the zero friction and zero inertia cases.

3. Extend the program to include a larger range of inertia ratios.

4. Reprogram the contact phase to include the case of elastic contact and to include the "lossy" case for all types of contact between elastic contact and plastic contact.

8. References

a. Dividing lines for backlash in the phaseplane, USNPGS thesis, 1959, William J. Lutkenhouse.

b. Characteristics of the Model 1604 Computer, Publication Number 018a, Control Data Corporation.

c. U.S.N.P.S. Programming Handbook for the 1604 Computer.

d. Preliminary Instruction Manual for Programming the Model 1604 Computer, Publication No. 015, Rev. 1, July 1, 1959, Control Data Corporation.

e. 1604 Assembly Routine "AR" (December 1959), Files of USNPGS Computer Center.

9. Tables

TABLE I

COMPUTER PROGRAM

```
                                REM         BACKLASH PROBLEM PLASTIC
                                             CONTACT RUN 3
                                ORG         20000
20000   75 4 20022   START      SLJ    4    INPUT 1     REMARKS IN THIS
        00 0 00000                0    0    0            COLUMN
20001   75 4 20063   COMBINED   SLJ    4    RESET
        00 0 00000                0    0    0
20002   75 4 60200              SLJ    4    RUNGE       SET UP GILL
        50 0 00000              ENI    0    0           (Procedure to solve
20003   00 0 20023                0    0    TABLE 1     differential
        00 0 20034                0    0    DERC        equations)
20004   75 4 60201   POINTC     SLJ    4    RUNGE +1
        00 0 00000                0    0    0
20005   75 4 60200   LOAD 1     SLJ    4    RUNGE       SET UP GILL
        50 0 00000              ENI    0    0
20006   00 0 20040                0    0    TABLE L
        00 0 20051                0    0    DERL
20007   75 4 60201   POINTL     SLJ    4    RUNGE +1
        00 0 00000                0    0    0
20010   75 4 20130              SLJ    4    COUNT
        50 0 00000              ENI    0    0
20011   75 0 20061              SLJ    0    TEST 1
        50 0 00000              ENI    0    0
20012   75 4 60200   MOTOR      SLJ    4    RUNGE       SET UP GILL
        50 0 00000              ENI    0    0
20013   00 0 20023                0    0    TABLE 1
        00 0 20054                0    0    DERM
20014   75 4 60201   POINTM     SLJ    4    RUNGE +1
        00 0 00000                0    0    0
20015   75 4 60200   LOAD2      SLJ    4    RUNGE       SET UP GILL
        50 0 00000              ENI    0    0
20016   00 0 20040                0    0    TABLE L
        00 0 20051                0    0    DERL
20017   75 4 60201   POINT2L    SLJ    4    RUNGE +1
        00 0 00000                0    0    0
20020   75 4 20130              SLJ    4    COUNT
        50 0 00000              ENI    0    0
20021   75 0 20073              SLJ    0    TEST 2
        00 0 00000                0    0    0
```

20022	75 0 00000	INPUT1	SLJ	0	0	Provision for changing
	75 0 20022		SLJ	0	INPUT1	Program if desired
20023	00 0 00000	TABLE1	DEC		2	Table for gill combined
	00 0 00002					
20024	17 7 15075		DEC		.01	
	34 1 21727					
20025	00 0 00000	T	DEC		0	
	00 0 00000					
20026	00 0 00000	UDOT	DEC		0	
	00 0 00000					
20027	00 0 00000	U	DEC		0	
	00 0 00000					
20030	00 0 00000	QU	DEC		0	
	00 0 00000					
20031	00 0 00000	THETADOT	DEC		0	
	00 0 00000					
20032	00 0 00000	THETA	DEC		0	
	00 0 00000					
20033	00 0 00000	QTHETA	DEC		0	
	00 0 00000					
20034	13 0 20156	DERC	LAC	0	B	Derivative for combined
	32 0 20027		FMU	0	U	
20035	31 0 20032		FSB	0	THETA	
	30 0 20164		FAD	0	ONE	
20036	20 0 20026		STA	0	UDOT	
	12 0 20027		LDA	0	U	
20037	20 0 20031		STA	0	THETADOT	
	75 0 60202		SLJ	0	RUNGE+2	
20040	00 0 00000	TABLEL	DEC		2	Table for gill load
	00 0 00002					
20041	17 7 15075		DEC		.01	
	34 1 21727					
20042	00 0 00000	TL	DEC		0	
	00 0 00000					
20043	00 0 00000	WDOT	DEC		0	
	00 0 00000					
20044	00 0 00000	W	DEC		0	
	00 0 00000					
20045	00 0 00000	QW	DEC		0	
	00 0 00000					
20046	00 0 00000	THETALD	DEC		0	
	00 0 00000					
20047	00 0 00000	THETAL	DEC		0	
	00 0 00000					

20050	00 0 00000	QTHETAL	DEC		0	
	00 0 00000					
20051	13 0 20157	DERL	LAC	0	D	Derivative for load
	32 0 20044		FMU	0	W	
20052	20 0 20043		STA	0	WDOT	
	12 0 20044		LDA	0	W	
20053	20 0 20046		STA	0	THETALD	
	75 0 60202		SLJ	0	RUNGE +2	
20054	13 0 20161	DERM	LAC	0	F	Derivative for motor
	32 0 20027		FMU	0	U	
20055	31 0 20047		FSB	0	THETAL	
	30 0 20164		FAD	0	ONE	
20056	33 0 20160		FDV	0	E	
	20 0 20026		STA	0	UDOT	
20057	12 0 20027		LDA	0	U	
	20 0 20031		STA	0	THETADOT	
20060	75 0 60202		SLJ	0	RUNGE +2	
	50 0 00000		ENI	0	0	
20061	12 0 20046	TEST1	LDA	0	THETALD	Test to determine
	31 0 20031		FSB	0	THETADOT	Start of backlash
20062	22 2 20012	JUMP1	AJP	2	MOTOR	
	75 0 20001		SLJ	0	COMBINED	
20063	75 0 00000	RESET	SLJ	0	0	
	12 0 20025		LDA	0	T	Changes load initial
20064	20 0 20042		STA	0	TL	conditions back to
	12 0 20026		LDA	0	UDOT	motor initial conditions
20065	20 0 20043		STA	0	WDOT	if separation has not
	12 0 20027		LDA	0	U	occurred
20066	20 0 20044		STA	0	W	
	12 0 20031		LDA	0	THETADOT	
20067	20 0 20046		STA	0	THETALD	
	12 0 20032		LDA	0	THETA	
20070	20 0 20047		STA	0	THETAL	
	10 0 00000		ENA	0	0	
20071	20 0 20045		STA	0	QW	
	20 0 20050		STA	0	QTHETAL	
20072	75 0 20063		SLJ	0	RESET	
	50 0 00000		ENI	0	0	
20073	12 0 20047	TEST2	LDA	0	THETAL	Test to determine if
	31 0 20032		FSB	0	THETA	backlash has been
20074	31 0 20166	BKTEST2	FSB	0	BACKLASH	taken up
	50 0 00000		ENI	0	0	
20075	22 2 20137	JUMP2	AJP	2	CONTACT	
	75 0 20012		SLJ	0	MOTOR	

TABLE I - Continued

20076	36 0 20117	SWITCH	SSK	0	ALTERNAT	Plan for changing
	75 0 20100		SLJ	0	LEFT	sign of backlash
20077	75 0 20107		SLJ	0	RIGHT	each cycle
	50 0 00000		ENI	0	0	
20100	12 0 20062	LEFT	LDA	0	JUMP 1	
	14 0 20116		ADD	0	FACTOR	
20101	20 0 20062		STA	0	JUMP 1	
	12 0 20075		LDA	0	JUMP 2	
20102	14 0 20116		ADD	0	FACTOR	
	20 0 20075		STA	0	JUMP 2	
20103	12 0 20074		LDA	0	BKTEST2	
	15 0 20172		SUB	0	FACTOR2	
20104	20 0 20074		STA	0	BKTEST2	
	12 0 20117		LDA	0	ALTERNAT	
20105	05 0 00001		ALS	0	1	
	20 0 20117		STA	0	ALTERNAT	
20106	75 0 20001		SLJ	0	COMBINED	
	50 0 00000		ENI	0	0	
20107	12 0 20062	RIGHT	LDA	0	JUMP1	
	15 0 20116		SUB	0	FACTOR	
20110	20 0 20062		STA	0	JUMP1	
	12 0 20075		LDA	0	JUMP2	
20111	15 0 20116		SUB	0	FACTOR	
	20 0 20075		STA	0	JUMP2	
20112	12 0 20074		LDA	0	BKTEST2	
	14 0 20172		ADD	0	FACTOR2	
20113	20 0 20074		STA	0	BKTEST2	
	12 0 20117		LDA	0	ALTERNAT	
20114	05 0 00001		ALS	0	1	
	20 0 20117		STA	0	ALTERNAT	
20115	75 0 20001		SLJ	0	COMBINED	
	50 0 00000		ENI	0	0	
20116	00 1 00000	FACTOR	OCT		0010000000000000	
	00 0 00000					
20117	25 2 52525	ALTERNAT	OCT		2525252525252525	
	25 2 52525					
20120	75 0 00000	PRINT	SLJ	0	0	Print routine
	12 0 20025		LDA	0	T	
20121	20 0 20252		STA	0	BUF	
	12 0 20027		LDA	0	U	Motor speed
20122	20 0 20253		STA	0	BUF+1	
	12 0 20032		LDA	0	THETA	Motor position

20123	20 0 20254		STA	0	BUF+2	
	12 0 20044		LDA	0	W	Load speed
20124	20 0 20255		STA	0	BUF+3	
	12 0 20047		LDA	0	THETAL	Load position
20125	20 0 20256		STA	0	BUF+4	
	75 4 71000		SLJ	4	DECO	
20126	01 0 20252		01	0	BUF	
	06 0 00001		06	0	1	
20127	72 0 20126		RAO	0	/-1	
	75 0 20120		SLJ	0	PRINT	
20130	75 0 00000	COUNT	SLJ	0	0	
	12 0 20170		LDA	0	INDEX	Plan for printing
20131	14 0 20171		ADD	0	INCRONE	only every tenth
	15 0 20173		SUB	0	TEN	point at .08 sec
20132	22 0 20134		AJP	0	OK2 PRINT	interval
	14 0 20173		ADD	0	TEN	
20133	20 0 20170		STA	0	INDEX	
	75 0 20130		SLJ	0	COUNT	
20134	75 4 20120	OK2PRINT	SLJ	4	PRINT	
	50 0 00000		ENI	0	0	
20135	10 0 00000		ENA	0	0	
	20 0 20170		STA	0	INDEX	
20136	75 0 20130		SLJ	0	COUNT	
	50 0 00000		ENI	0	0	
20137	12 0 20162	CONTACT	LDA	0	G	Apply law of
	32 0 20027		FMU	0	U	momentum
20140	20 0 20165		STA	0	GU	
	12 0 20163		LDA	0	H	
20141	32 0 20044		FMU	0	W	
	30 0 20165		FAD	0	GU	
20142	20 0 20031		STA	0	THETADOT	Puts resulting values
	20 0 20027		STA	0	U	of variables in
20143	20 0 20044		STA	0	W	Table 1 and L
	20 0 20046		STA	0	THETALD	
20144	13 0 20156		LAC	0	B	
	32 0 20027		FMU	0	U	
20145	31 0 20032		FSB	0	THETA	
	30 0 20164		FAD	0	ONE	
20146	20 0 20026		STA	0	UDOT	
	13 0 20157		LAC	0	D	
20147	32 0 20044		FMU	0	W	
	20 0 20043		STA	0	WDOT	

20150	12 0 20047		LDA	0	THETAL	
	20 0 20032		STA	0	THETA	
20151	10 0 00000		ENA	0	0	
	20 0 20030		STA	0	QU	
20152	20 0 20033		STA	0	QTHETA	
	20 0 20045		STA	0	QW	
20153	20 0 20050		STA	0	QTHETAL	
	75 4 20120		SLJ	4	PRINT	
20154	50 0 00000		ENI	0	0	
	75 0 20076		SLJ	0	SWITCH	
20155	50 0 00000		ENI	0	0	
	50 0 00000					
20156	17 7 66314	B	DEC		.4	B equals twice the
	63 1 46314					damping coefficient
20157	17 7 54000	D	DEC		.125	D equals the load
	00 0 00000					time constant
20160	17 7 56314	E	DEC		.2	E equals motor inertia
	63 1 46314					
20161	17 7 64631	F	DEC		.3	F equals motor friction
	46 3 14631					
20162	17 7 56314	G	DEC		.2	G equals motor inertia
	63 1 46314					
20163	20 0 06314	H	DEC		.8	H equals load inertia
	63 1 46314					
20164	20 0 14000	ONE	DEC		1.	Integer
	00 0 00000					
20165	00 0 00000	GU	DEC		0	Constant initially zero
	00 0 00000					
20166	17 7 64631	BACKLASH	DEC		.3	Amount of backlash
	46 3 14631					
20167	01 0 00000	FACTOR2	OCT	0100000000000000		
	00 0 00000					
20170	00 0 00000	INDEX	OCT		0	
	00 0 00000					
20171	00 0 00000	INCRONE	OCT		1	Used in count
	00 0 00001					
20172	00 0 00000	BLANK	OCT		0	
	00 0 00000					
20173	00 0 00000	TEN	OCT		10	Used in count
	00 0 00010					
20174	17 7 15075	1	DEC		.01	Beginning of floating-
	34 1 21727					point conversion table

20175	17 7 25075 34 1 21727	2	DEC	.02
20176	17 7 27534 12 1 72702	3	DEC	.03
20177	17 7 35075 34 1 21727	4	DEC	.04
20200	17 7 36314 63 1 46314	5	DEC	.05
20201	17 7 37534 12 1 72702	6	DEC	.06
20202	17 7 44365 60 5 07534	7	DEC	.07
20203	17 7 45075 34 1 21727	8	DEC	.08
20204	17 7 45605 07 5 34121	9	DEC	.09
20205	17 7 46314 63 1 46314	10	DEC	.10
20206	17 7 47024 36 5 60507	11	DEC	.11
20207	17 7 47534 12 1 72702	12	DEC	.12
20210	17 7 54121 72 7 02436	13	DEC	.13
20211	17 7 54365 60 5 07534	14	DEC	.14
20212	17 7 54631 46 3 14631	15	DEC	.15
20213	17 7 55075 34 1 21727	16	DECq	.16
20214	17 7 55341 21 7 27024	17	DEC	.17
20215	17 7 55605 07 5 34121	18	DEC	.18
20216	17 7 56050 75 3 41217	19	DEC	.19
20217	17 7 56314 63 1 46314	20	DEC	.20
20220	17 7 57024 36 5 60507	22	DEC	.22
20221	17 7 57534 12 1 72702	24	DEC	.24
20222	17 7 64000 00 0 00000	25	DEC	.25

20223	17 7 64121	26	DEC	.26
	72 7 02436			
20224	17 7 64365	28	DEC	.28
	60 5 07534			
20225	17 7 64631	30	DEC	.30
	46 3 14631			
20226	17 7 65075	32	DEC	.32
	34 1 21727			
20227	17 7 65463	35	DEC	.35
	14 6 31463			
20230	17 7 65605	36	DEC	.36
	07 5 34121			
20231	17 7 66314	40	DEC	.40
	63 1 46314			
20232	17 7 67024	44	DEC	.44
	36 5 60507			
20233	17 7 67146	45	DEC	.45
	31 4 63146			
20234	17 7 67534	48	DEC	.48
	12 1 72702			
20235	20 0 04000	50	DEC	.50
	00 0 00000			
20236	20 0 04314	55	DEC	.55
	63 1 46314			
20237	20 0 04631	60	DEC	.60
	46 3 14631			
20240	20 0 05146	65	DEC	.65
	31 4 63146			
20241	20 0 05463	70	DEC	.70
	14 6 31463			
20242	20 0 06314	80	DEC	.80
	63 1 46314			
20243	20 0 07146	90	DEC	.90
	31 4 63146			
20244	20 0 14000	100	DEC	1.00
	00 0 00000			
20245	20 0 14631	120	DEC	1.20
	46 3 14631			
20246	20 0 15463	140	DEC	1.40
	14 6 31463			
20247	20 0 16314	160	DEC	1.60
	63 1 46314			

```
20250     20 0 17146     180        DEC        1.80
          31 4 63146
20251     20 0 24000     200        DEC        2.00
          00 0 00000
20252     00 0 00000     BUF        BSS        6
          00 0 00000
                         DECO       EQU        71000
                         RUNGE      EQU        60200

                                    END
```

TABLE II

TYPICAL PRINTOUT OF DATA

TIME	MOTOR SPEED	MOTOR POSITION	LOAD SPEED	LOAD POSITION
79999999- 1	77040630- 1	30333708- 2	68901844- 1	30007982- 2
15999999	15278417	12247439- 1	14489345	12215856- 1
23999999	22518046	27389548- 1	21757968	27359124- 1
31999999	29387916	48177680- 1	28660703	48148568- 1
39999999	35856424	74303153- 1	35165619	74275496- 1
47999999	41895488	10543340	41244294	10540733
55999999	47480616	14121485	46871885	14119047
63999999	52590944	18127574	52027169	18125317
71999999	57209252	22522912	56692568	22520843
79999999	61321964	27267569	60854140	27265695
87999999	64919110	32320675	64501553	32319002
95999999	67994283	37640704	67628038	37639237
10399999 1	70544563	43185760	70230320	43184500
11199999 1	72570424	48913847	72308523	48912797
11999999 1	74075629	54783141	73866071	54782300
12799999 1	75067101	60752245	74909556	60751613
13599999 1	75554775	66780440	75448598	66780013
14399999 1	75551450	72827914	75495690	72827688
15199999 1	75072608	78855989	75066030	78855961
15999999 1	73561932	84814975	74326600	84831950
16799999 1	70007602	90570699	73587038	90748447
17599999 1	64634564	95967717	72854835	96606073
18399999 1	57670572	10086982 1	72129917	10240541 1
19199999 1	49317406	10515798 1	71412213	10814705 1
19999999 1	39753812	10872835 1	70701649	11383155 1
20799999 1	29138106	11149055 1	69998156	11945950 1
21599999 1	17610490	11336614 1	69301663	12503145 1
22399999 1	52951014- 1	11428723 1	68612100	13054795 1
23199999 1	-76981710- 1	11419529 1	67929398	13600957 1
23999999 1	-21272070	11304005 1	67253489	14141684 1
24199999 1	48719261	14276023 1	48719261	14276023 1
24799999 1	44993108	14557350 1	45474833	14557542 1
25599999 1	39850916	14896797 1	40349320	14896996 1
26399999 1	34619795	15194727 1	35131214	15194932 1
27199999 1	29335478	15450572 1	29856282	15450780 1
27999999 1	24032905	15664046 1	24559521	15664257 1
28799999 1	18746033	15835140 1	19274963	15835352 1
29599999 1	13507650	15964112 1	14035491	15964323 1

Flow Diagram For Backlash Digital Program

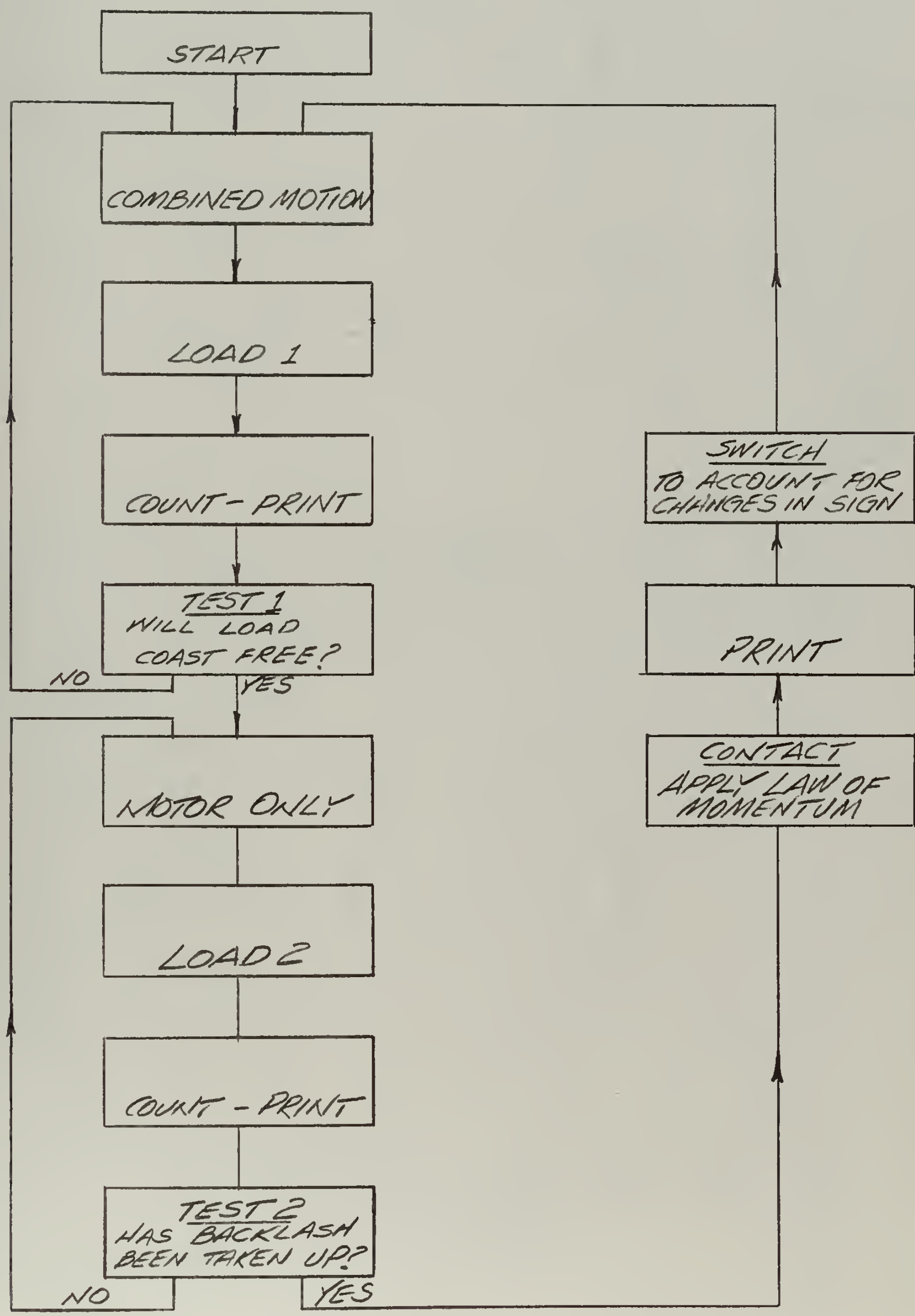
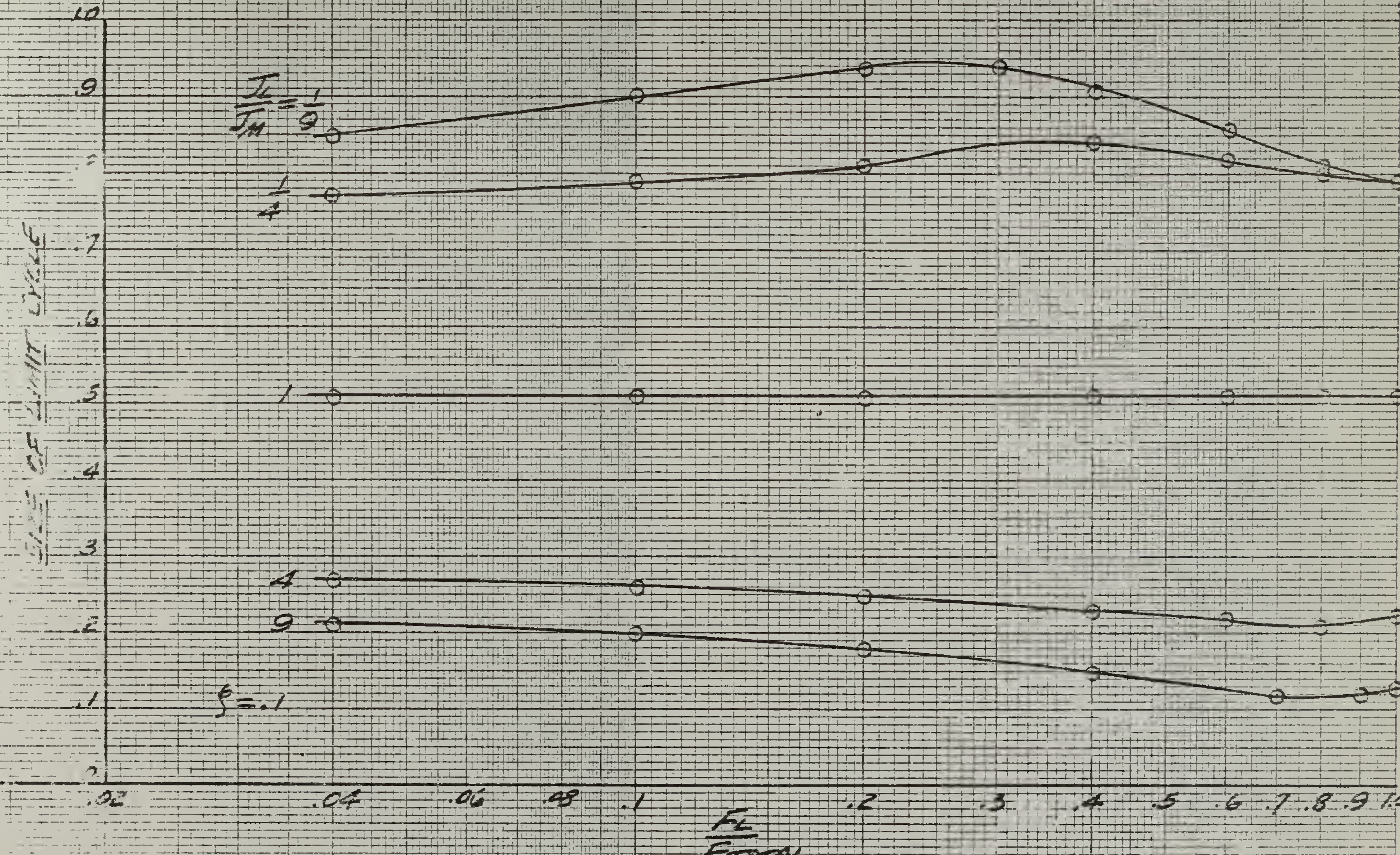


FIGURE 1



SIZE OF LIMIT CYCLE FOR DAMPING COEFFICIENT EQUAL TO 0.1 AND BACKLASH EQUAL TO 0.3.

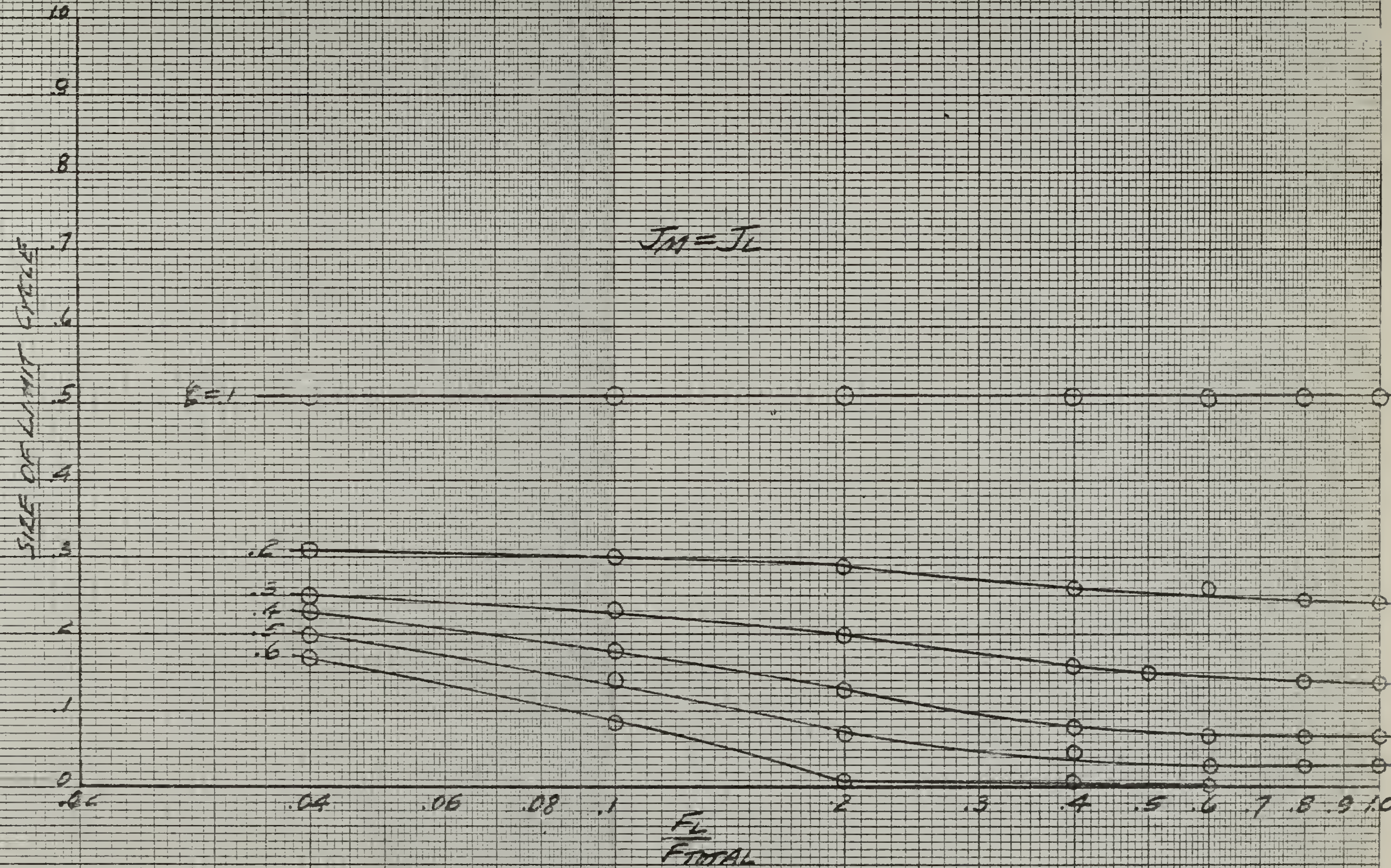


FIGURE 3. SIZE OF LIMIT CYCLE FOR BALANCED INERTIA AND BACKLASH EQUAL TO 0.3

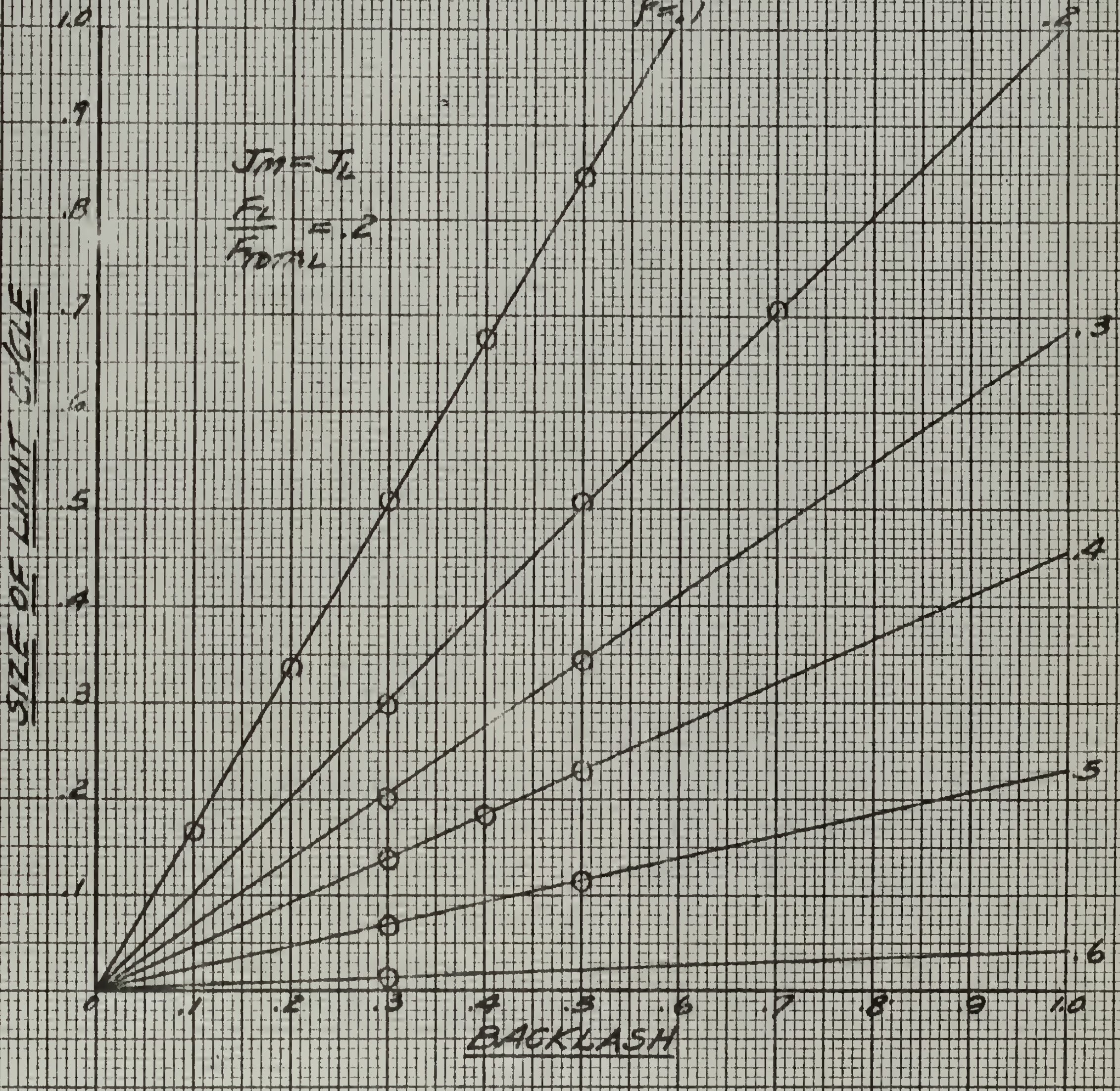


FIGURE 4. SIZE OF LIMIT CYCLE FOR BALANCED INERTIA AND FRICTION RATIO OF 0.2

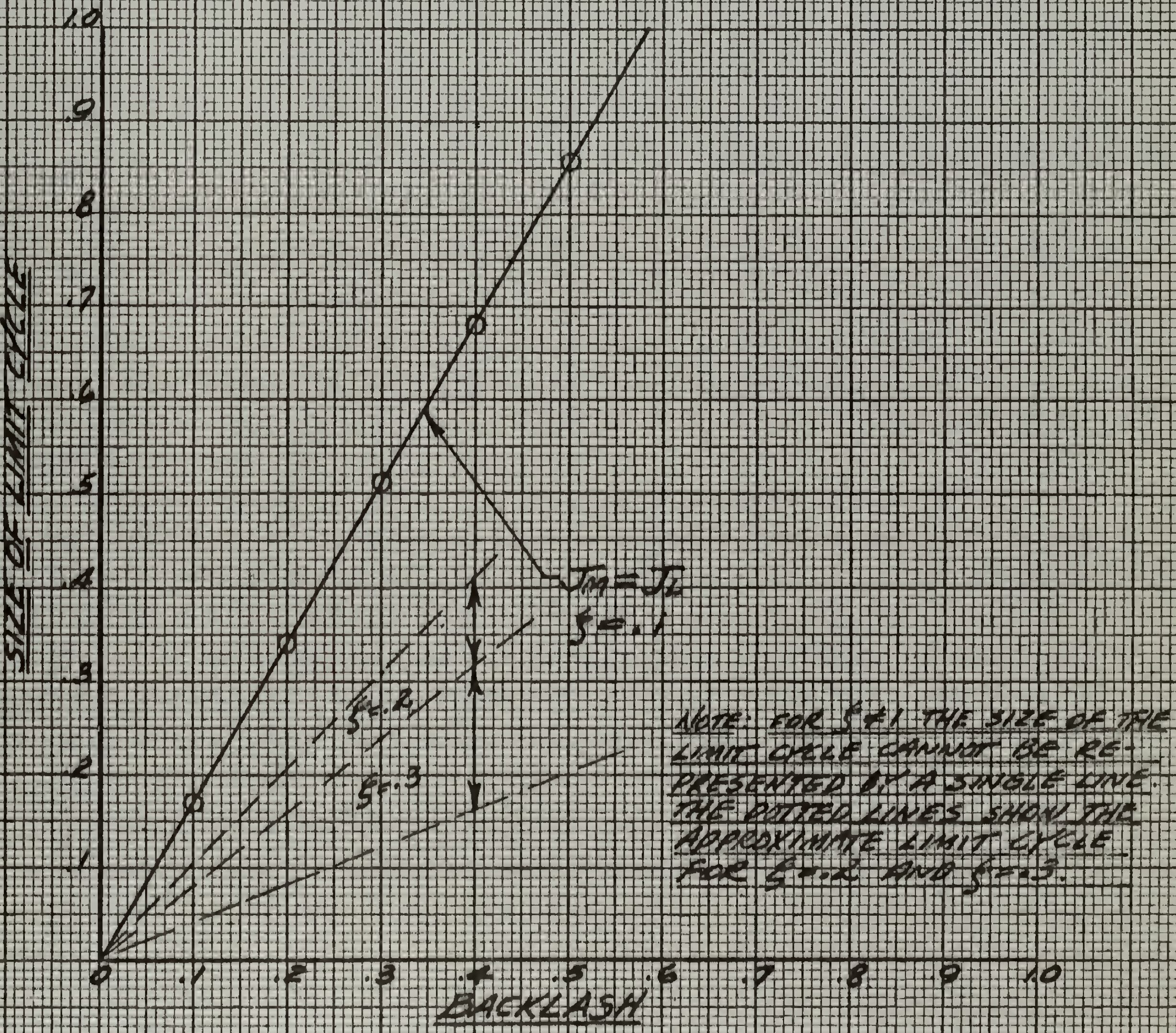


FIGURE 5. SIZE OF LIMIT CYCLE FOR BALANCED INERTIA AND FRICTION RATIO EQUAL TO ALL VALUES. DAMPING COEFFICIENT IS EQUAL TO 0.1. SEE NOTE.

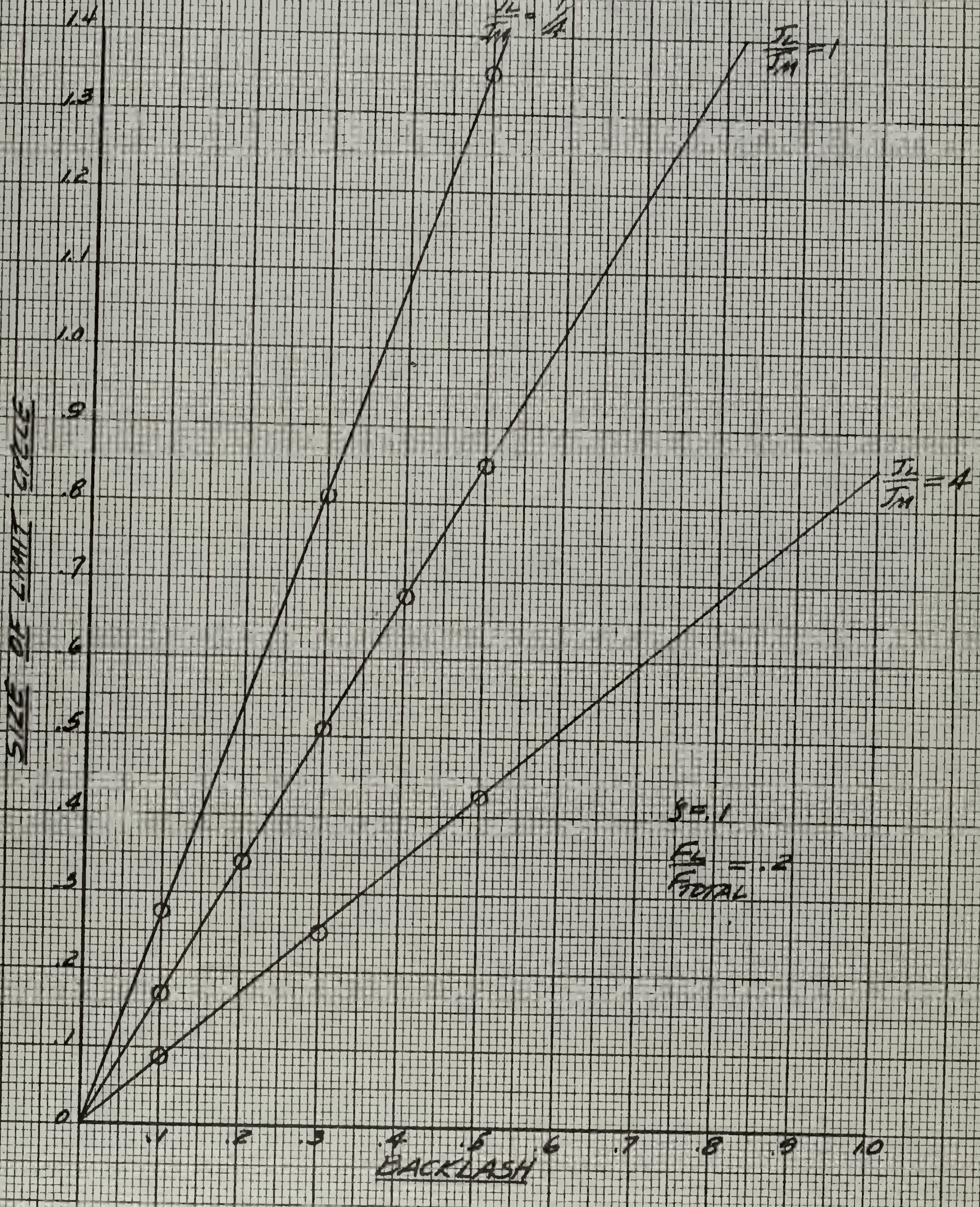


FIGURE 6. SIZE OF LIMIT CYCLE FOR FRICTION RATIO OF 0.2 AND DAMPING COEFFICIENT OF 0.1

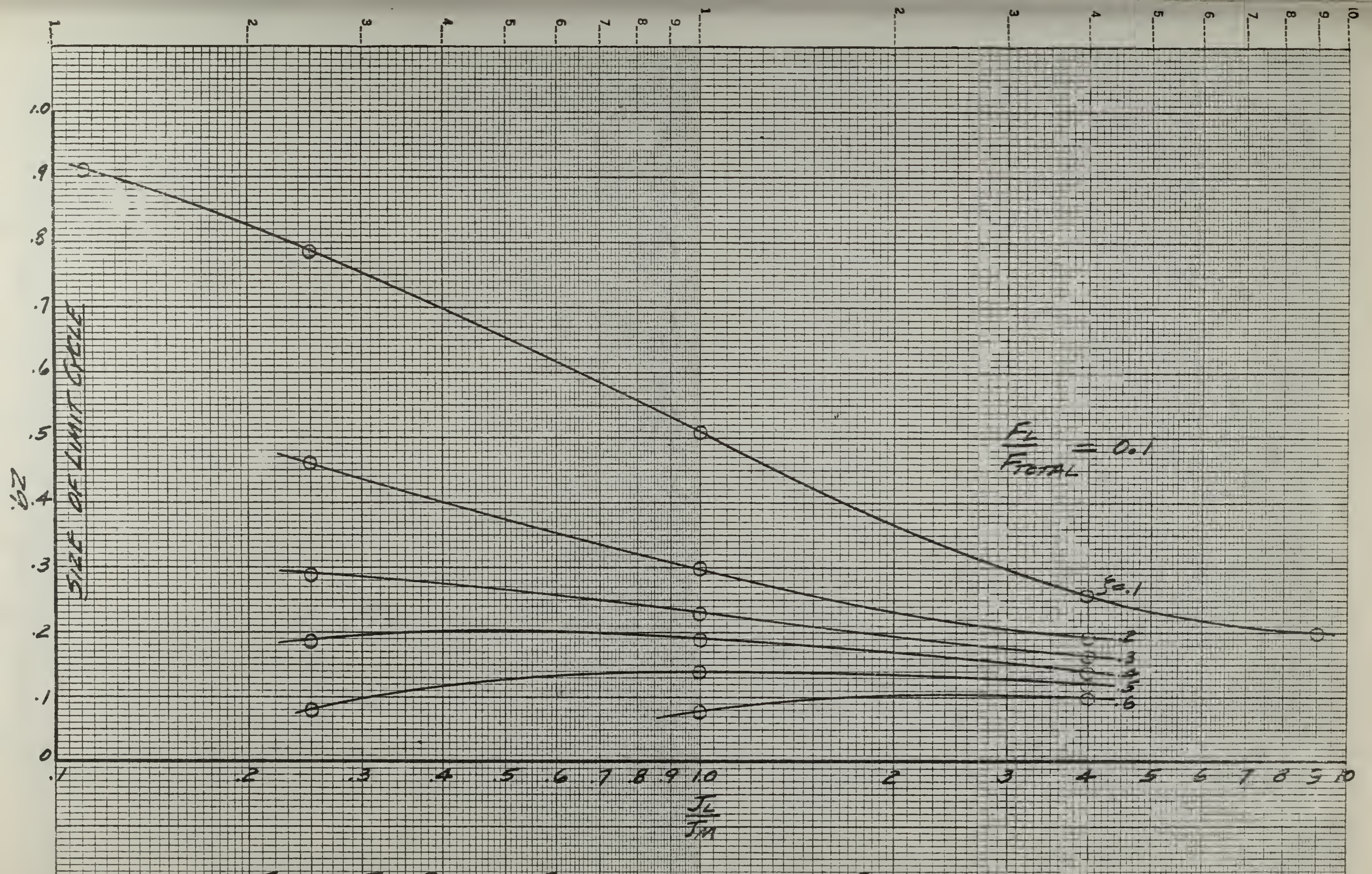


FIGURE 7. DESIGN CHART FOR FRICTION RATIO OF 0.1 AND BACKLASH EQUAL TO 0.3

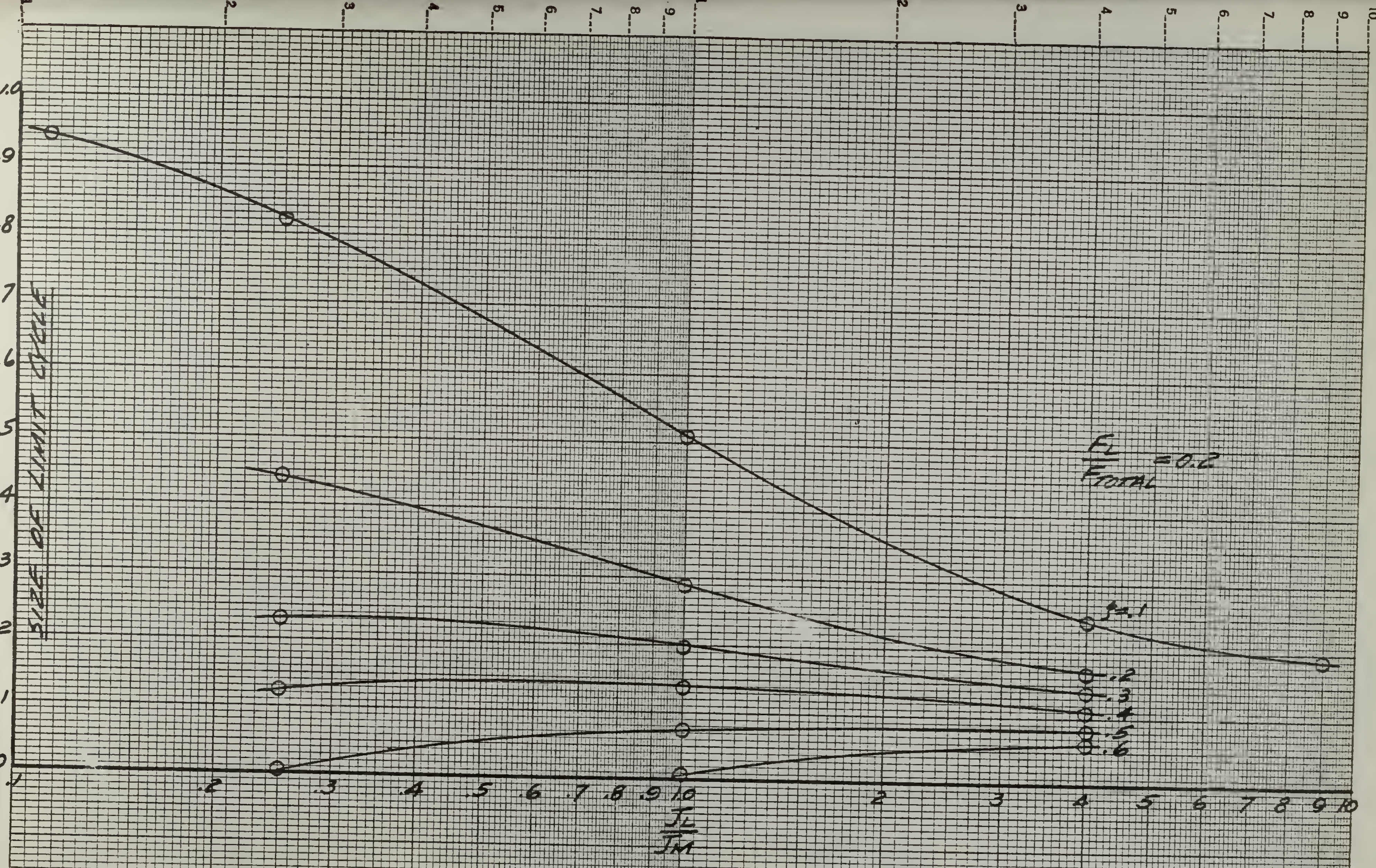


FIGURE 8. DESIGN CHART FOR FRICTION RATIO OF 0.2 AND BACKLASH EQUAL TO 0.3

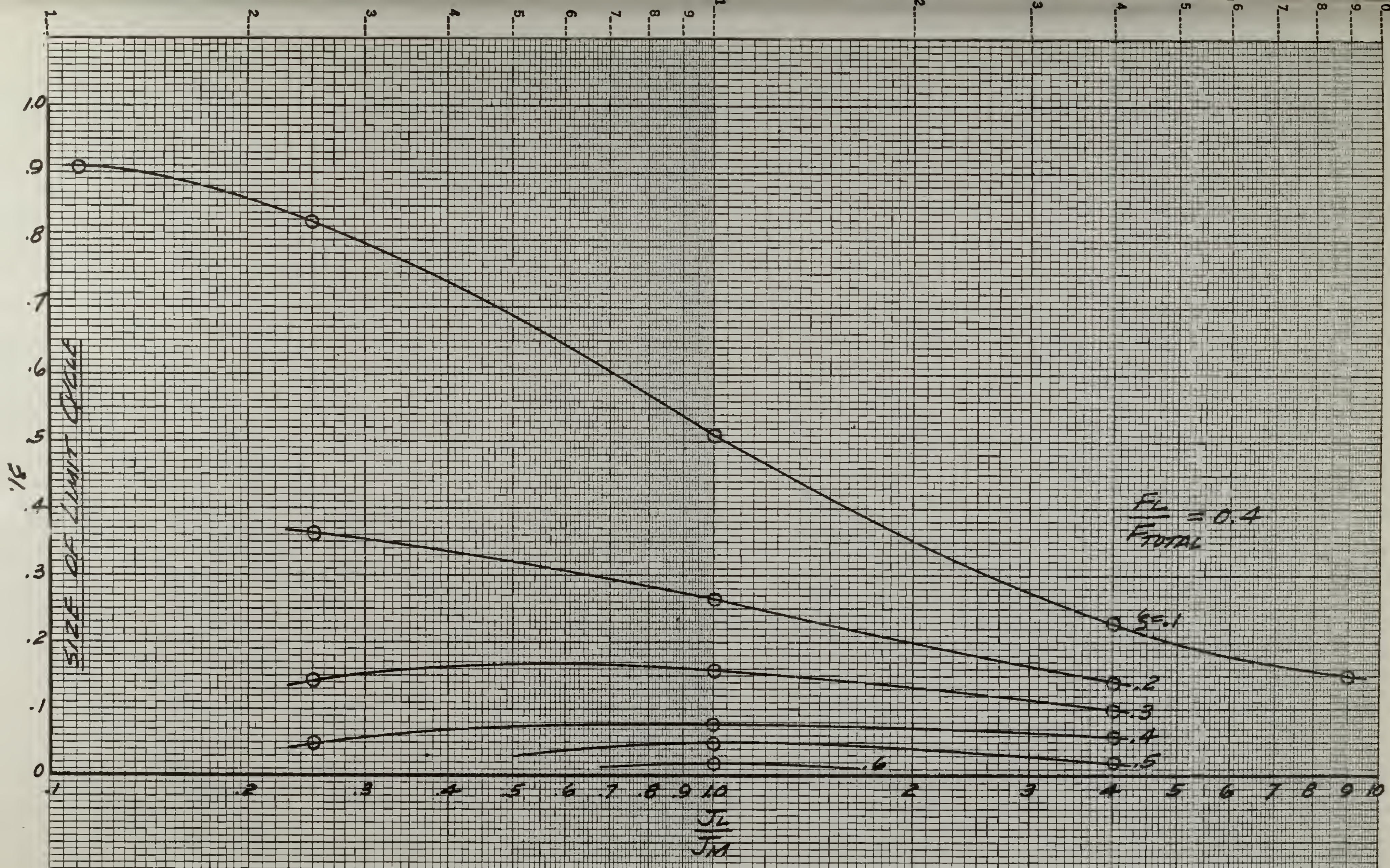


FIGURE 9. DESIGN CHART FOR FRICTION RATIO OF 0.4 AND BACKLASH EQUAL TO 0.3

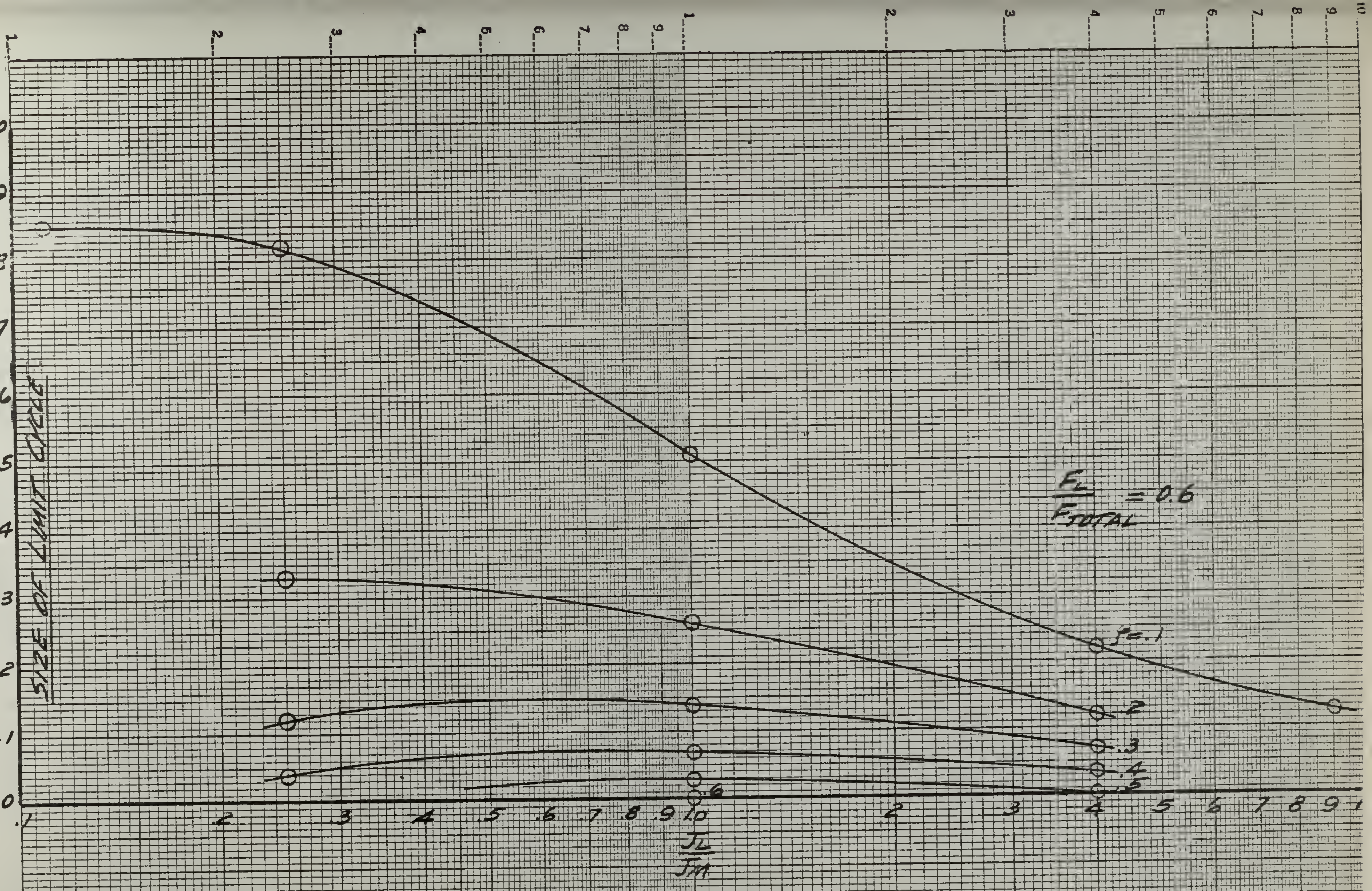


FIGURE 10. DESIGN CHART FOR FRICTION RATIO OF 0.6 AND BACKLASH EQUAL TO 0.3

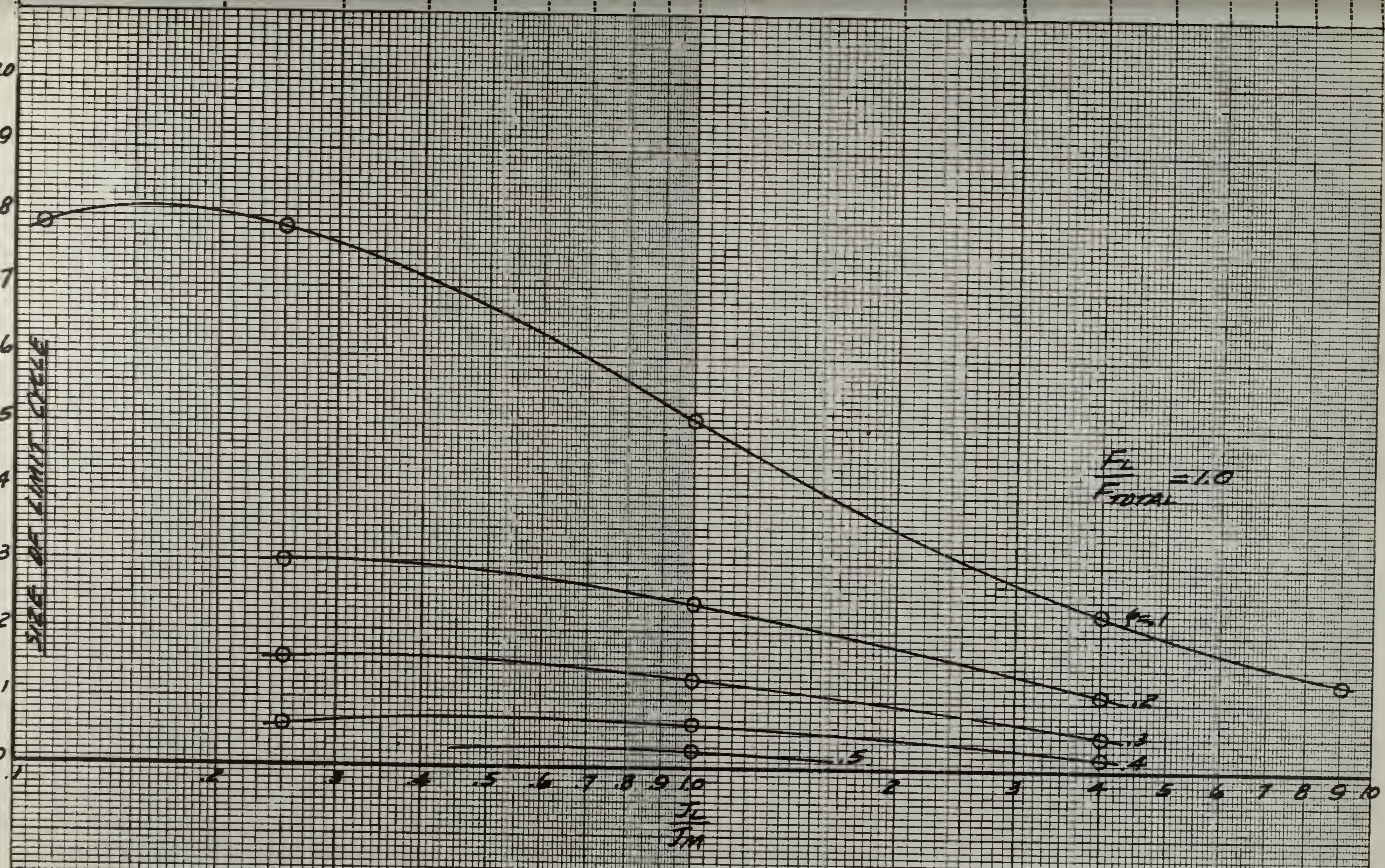
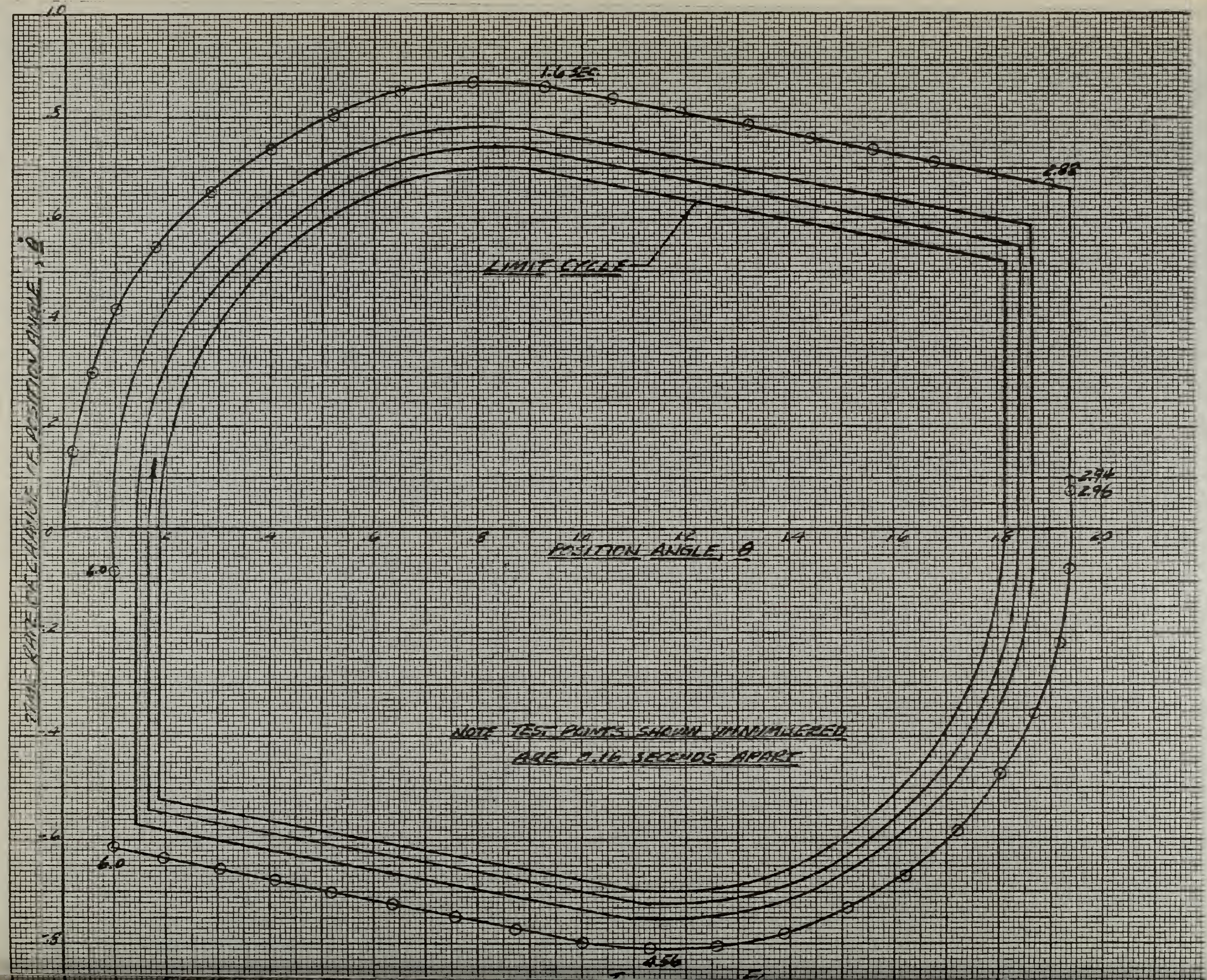


FIGURE 11. DESIGN CHART FOR FRICTION RATIO OF 1.0 AND BACKLASH EQUAL TO 0.3



TIME RATE OF CHANGE OF POSITION ANGLE, $\dot{\theta}$
POSITION ANGLE, θ
LIMIT CYCLE
NOTE TEST POINTS SHOWN UNNUMBERED ARE 0.16 SECONDS APART
1.6 SEC.
2.88
2.94
2.96
6.0
6.0
4.56
1.0
.8
.6
.4
.2
0
.2
.4
.6
.8
.2
.4
.6
.8
1.0
1.2
1.4
1.6
1.8
2.0

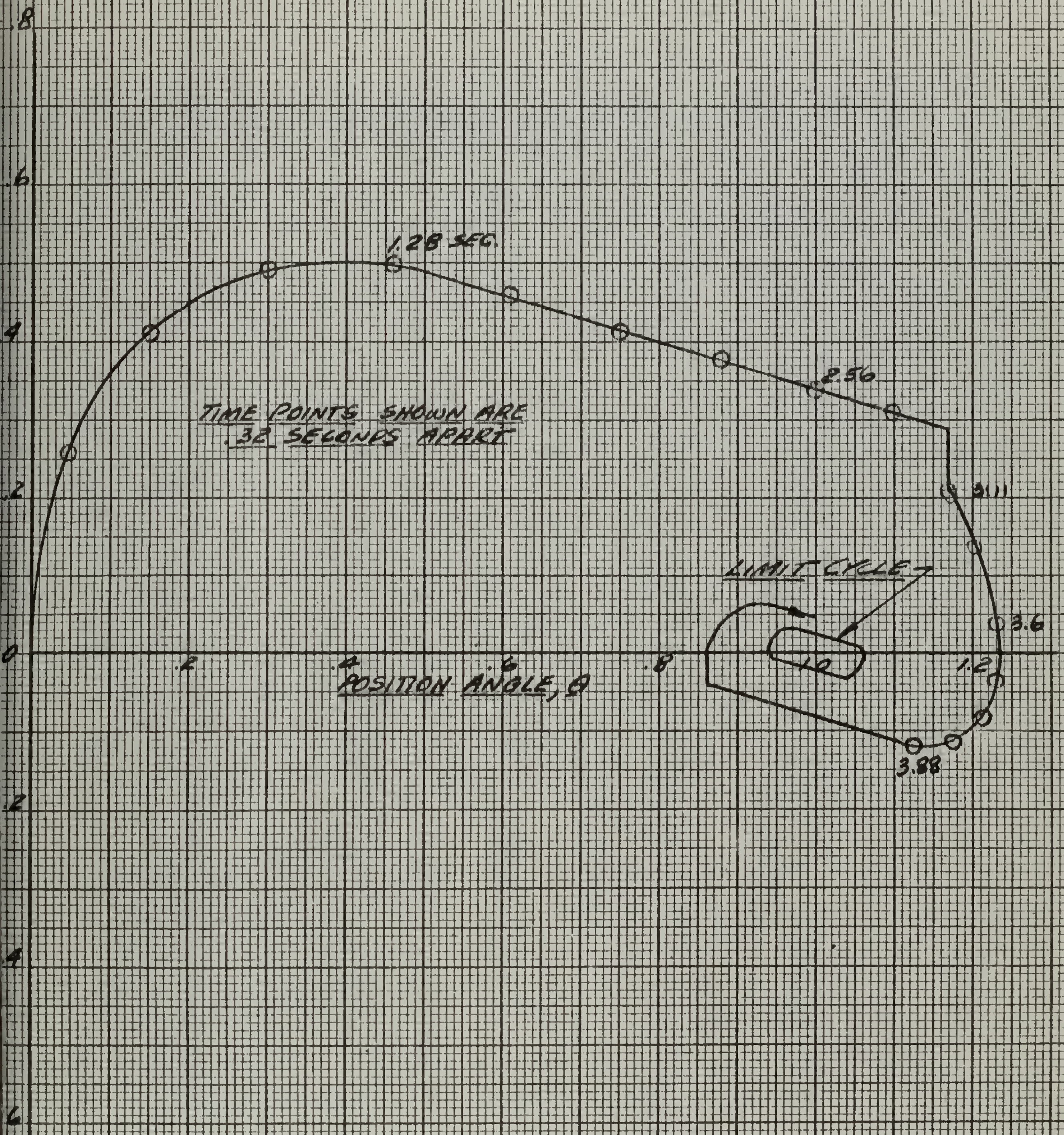


FIGURE 13. PHASE TRAJECTORY FOR $\frac{J_L}{J_M} = 4$ $\frac{F_L}{F_{TOTAL}} = .2$ $\zeta = .6$ AND BACKLASH = .3

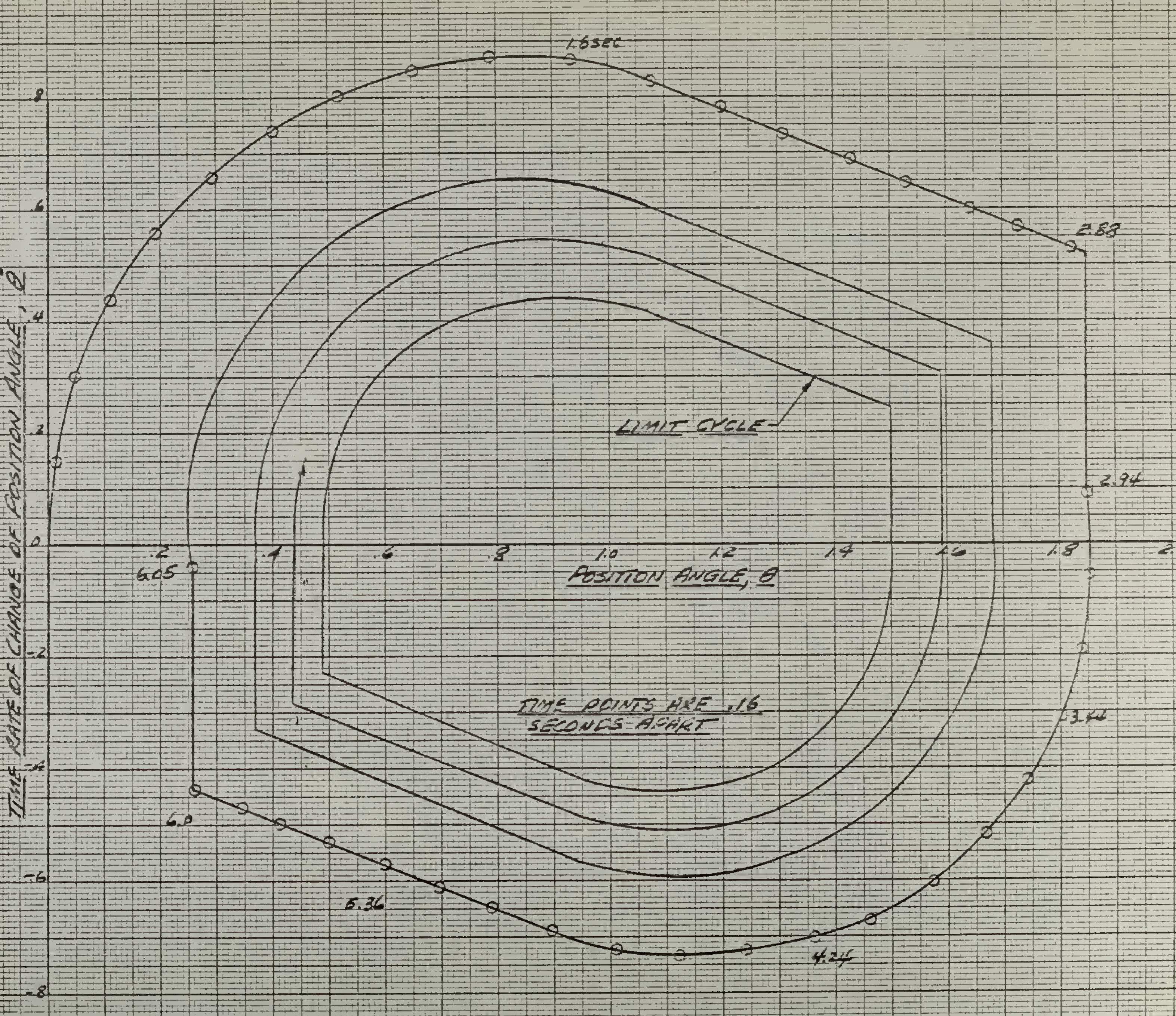


FIGURE 14. PHASE TRAJECTORY FOR $\frac{T_L}{T} = 1.0$, $\frac{F_C}{F} = 1.0$

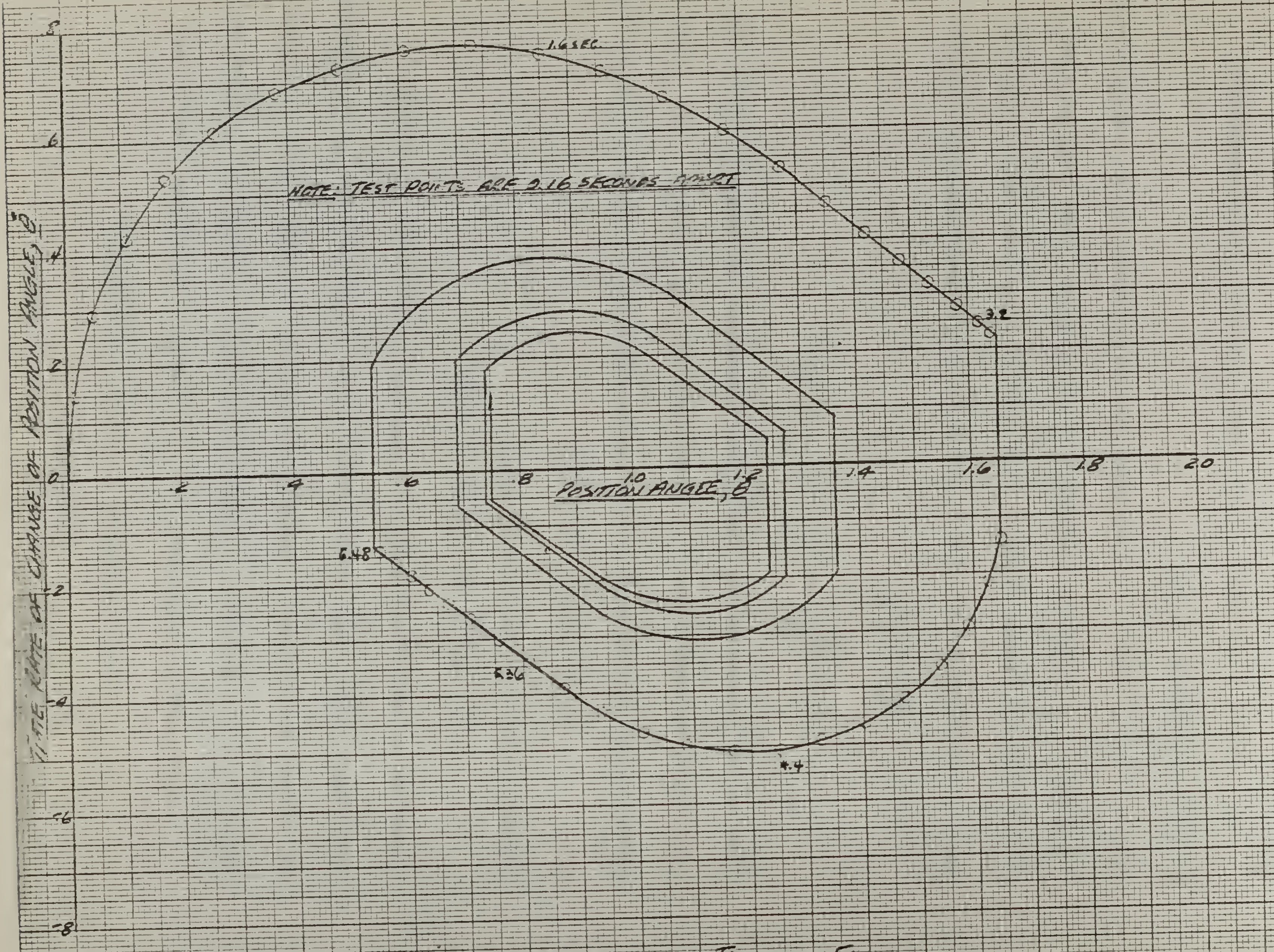


FIGURE 15. PHASE TRAJECTORY FOR $\frac{J_L}{J_M} = 1.0$ $\frac{F_L}{F_{TOTAL}} = 1.0$
$\zeta = .2$ AND BACKLASH $= .3$

APPENDIX A

DEVELOPMENT OF CHART PARAMETERS

The purpose of this appendix is to present a procedure whereby the factors required to enter the design charts of this thesis may be computed for a given servo system. Equations for the field control motor, the armature control motor, and the two phase motor are developed. The application of a given system represented by a differential equation of the form

$$\ddot{\theta} + 2\zeta\omega_n\dot{\theta} + \omega_n^2\theta = \omega_n^2\theta_R$$

requires a transformation to the form

$$\ddot{\theta} + 2\zeta\dot{\theta} + \theta = \theta_R$$

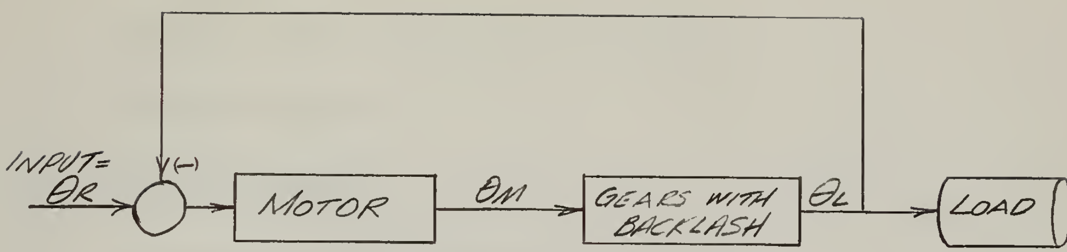
since the design charts are based on an undamped natural frequency of one. This can be accomplished by a time scaling of the problem. Thus, in the first equation above if the independent variables are related by $\omega_n t = t'$

where $$\dot{\theta} = \frac{d\theta}{dt} = \frac{d\theta}{dt'}\frac{dt'}{dt} = \dot{\theta}\omega_n$$

and $$\ddot{\theta} = \ddot{\theta}\omega_n^2$$

When these substitutions are made there results the desired form suitable for use with the design charts. From this, it is evident that the damping coefficient, ζ , remains unaltered in the transformation, and the design charts, while based on an undamped natural frequency of one, are applicable to other frequencies as well.

The second order servo can be represented by a block diagram as follows.



Three types of motors are considered, the armature control motor, the field control motor, and the two phase motor. For the period of time when the gears are in contact the system is a simple second order system. The differential equations for this combined motion can be derived by equating the load torque required to the motor torque developed. For the armature control motor this results in the following equation.

$$\ddot{\theta} + \frac{F_T + F_1}{J_M + J_L}\dot{\theta} + \frac{K}{J_M + J_L}\theta = \frac{K}{J_M + J_L}\theta_R$$

For the field control motor if the field time constant is negligible the equation is similar.

$$\ddot{\theta} + \frac{F_T}{J_M + J_L}\dot{\theta} + \frac{K}{J_M + J_L}\theta = \frac{K}{J_M + J_L}\theta_R$$

In like manner the differential equation for a two phase motor is

$$\ddot{\theta} + \frac{F_T + K_N}{J_M + J_L}\dot{\theta} + \frac{K_e}{J_M + J_L}\theta = \frac{K_e}{J_M + J_L}\theta_R$$

where

K_1 = motor torque constant

K_2 = motor generator constant

K_3 = error measurement constant

R = armature resistance

ρ = gear constant

J_M = inertia of motor referred to load shaft

J_m = inertia of motor

J_L = inertia of load

f_m = friction constant of motor

F_M = friction constant of motor referred to load shaft

F_L = friction constant of load

F_T = friction of load plus motor friction referred to load shaft

$K = \dfrac{K_1 K_3}{\rho R}$

$F_1 = \dfrac{K_1 K_2}{\rho^2 R}$

$K_N = -\dfrac{\partial T}{\partial n}$ (from characteristic curves of two phase motor)

$K_e = \dfrac{\partial T}{\partial e_1}$ (from characteristic curves of two phase motor)

Procedure for Armature Control Motor

1. Find K from $K = \frac{K_1 K_3}{PR}$

2. Find J_M referred to load shaft.

3. Find $\omega_n = \sqrt{\frac{K}{J_M + J_L}}$

4. Find F_T from $F_T = F_L + \frac{f_m}{P^2}$

5. Find F_1 from $F_1 = \frac{K_1 K_2}{P^2 R}$

6. Find ζ $\zeta = \left(\frac{F_T + F_1}{J_M + J_L}\right)\left(\frac{1}{2\omega_n}\right)$

7. Find J_L/J_M

8. Find $F_L/F_{Total} = \frac{F_L}{F_T + F_1}$

9. Use 6, 7, and 8 to select proper design chart and find size of limit cycle for 0.3 rad backlash.

10. Find size of limit cycle for desired backlash from

$$\text{LIMIT CYCLE} = (\text{CHART LIMIT CYCLE})\left(\frac{\text{BACKLASH}}{0.3}\right)$$

Procedure for Field Control Motor

1. through 4. Same as Armature Control Motor

5. Find ζ $\zeta = \left(\frac{F_T}{J_M + J_L}\right)\left(\frac{1}{2\omega_n}\right)$

6. Continue as Armature Control Motor

Procedure for Two Phase Motor

1. Find K_e from characteristic curves of motor

2. Find J_M referred to load chart

3. Find $\omega_n = \sqrt{\frac{K_e}{J_M + J_L}}$

4. Find F_{total}

5. Find K_n from characteristic curves of motor.

6. Find ζ $\quad \zeta = \left(\frac{F_T + K_N}{J_M + J_L}\right)\left(\frac{1}{2\,\omega_n}\right)$

7. Continue as Armature Control Motor.

APPENDIX B

SAMPLE PROBLEM

A Elinco FD-162 115 volt D. C. Shunt Motor, 1/125 Horsepower 4,000 Rpm is coupled through a gearbox to a load. The constants of the proportional error servo are as follows:

Armature resistance, $R = 282$ ohms

Motor Generator constant, $K_2 = 0.233$ volts/rad/sec

Torque Constant, $K_1 = 28.2$ ounce - inch/ampere

Motor Inertia, $J_m = 0.61$ ounce-inch^2

Motor Friction, $f_m = .00064$ ounce-inch/rad/sec

Error measurement constant, $K_3 = 10$ volts/rad

Gearbox factor, $\rho = 1$

Backlash 0.1 rad

Load Inertia $J_L = 16.4 \times 10^{-6}$ slug ft^2

Load Friction $F_L = .002$ ounce-inch/rad/sec

<u>Solution</u>: Using the procedure for the armature control motor in appendix A.

1. $K = \frac{K_1 K_2}{\rho R} = \frac{(28.2)(10)}{(16)(12)(1)(282)} = .0052$ FT LB.

2. $J_M = \frac{J_m}{\rho^2} = \frac{0.61}{(16)(12)^2(32.2)} = 8.2 \times 10^{-6}$ SLUG FT^2

3. $\omega_n = \sqrt{\frac{K}{J_M + J_L}} = \sqrt{\frac{.0052}{24.6 \times 10^{-6}}} = 14.5$ Rad/sec

4. $$F_T = F_L + \frac{f_m}{P^2} = \frac{.002 + .00064}{(16)(12)} = 13.7 \times 10^{-6} \text{ FT LB/RAD/SEC}$$

5. $$F_1 = \frac{K_1 K_2}{P^2 R} = \frac{(28.2)(.233)}{(12)(16)(1)^2(282)} = 121 \times 10^{-6} \text{ FT LB/RAD/SEC}$$

6. $$\zeta = \left(\frac{F_T + F_1}{J_M + J_L}\right)\left(\frac{1}{2\omega_n}\right) = \frac{135 \times 10^{-6}}{(24.6 \times 10^{-6})(2)(14.5)} = .19$$

7. $$\frac{J_L}{J_M} = \frac{16.4 \times 10^{-6}}{8.2 \times 10^{-6}} = 2.0$$

8. $$\frac{F_L}{F_{TOTAL}} = \frac{.002}{(12)(16)(135 \times 10^{-6})} = .077$$

9. From Figure 7 representing a friction ratio of .1, and for an inertia ration of 2.0 and a damping coefficient of .19 the size of the limit cycle for 0.3 radian backlash is .25 radian.

10. The size of the limit cycle for 0.1 radian backlash is

$$\text{Limit cycle} = .25 \times \frac{(.1)}{(.3)} = \underline{\underline{.083 \text{ Radian.}}}$$

APPENDIX C

USE OF DESIGN CHARTS

Figures 7 through 11 are presented as representing design charts for the second order servo with backlash. These charts can be used to rapidly determine the size of the limit cycle by the following procedure.

1. If the system variables are not known, compute these by the methods of Appendix A.
2. Choose the chart representing the Load Friction/Total Friction desired.
3. Read the Size of the Limit Cycle resulting from the intersection of the abscissa representing inertia ratio and the desired damping coefficient, interpolating if necessary.
4. The value determined represents that Limit Cycle resulting from a backlash of 0.3 radians. For a given backlash the limit cycle is directly proportional, thus divide the Limit Cycle found in 3 above by 0.3 and multiply by the desired backlash.